MATHEMATICAL MODELING OF A ROTARY VACUUM WASHER USED FOR PULP WASHING: A CASE STUDY OF A LAB SCALE WASHER

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Mathematical models for the different zones of a rotary vacuum washer are presented in terms of their fundamental parameters. Filter performance is expressed in terms of washing liquor usage, solute removal and efficiency parameters. Laboratory experiments are performed on a combination of Indian hardwood (eucalyptus, bamboo and pine) pulp collected from a paper mill. The influence of interstitial velocity, cake thickness, Peclet number, bed porosity and dilution factor on the commonly used industrial parameters, such as washing efficiency, displacement ratio, Norden efficiency factor, is examined. The proposed model can be used in developing computer codes for online monitoring of the process.

Keywords: axial dispersion coefficient, consistency, cake thickness, dilution factor, porosity, displacement ratio, washing efficiency, interstitial velocity

INTRODUCTION

Plant washing is a link between the pulping and the recovery section, separating the solute contained in the stagnant areas between fibers, in fiber lumen and fiber walls, thus producing clean pulp for further treatments. An efficient pulp treatment results in lower soda loss, less carryover of the black liquor solids, reduced volume of the spent liquor and lower cost of the black liquor treatment. Ideal conditions, *i.e.* no overflow of the black liquor solids with washed pulp and no overflow of fibers with the filtrate, can be never obtained industrially. Hence, the pulp should be optimally washed with minimum amounts of wash water and maximum removal of black liquor solutes.

The rotary vacuum washer, largely preferred all over the globe, consists of a wire mesh covered cylinder which rotates in the vat containing the slurry. A pulp mat is formed on the outer surface of the cylinder, as due to the vacuum inside the cylinder. On the upper part of the cylinder, the wash water is applied. The rotary drum comprises cake formation, first dewatering, washing, second dewatering, blow and discharge, and dead zones (Fig. 1). The total area combining cake formation and washing zones is of nearly 50%. Both the thick and the thin filtrates from these zones contribute considerably to the total filtrate. The first and second dewatering zones are engaged in discharging much lower amounts of filtrate. Besides the liquor, the air is also extracted through these zones, exhibiting a two-phase flow situation. The contribution of the air to the black liquor displacement is not considerable. The dead and the discharge zones have practically no contribution to filtrate generation. The mechanics of the washing process involves the displacement of the liquor by the movement of the water plug controlled by fluid mechanics, dispersion due to back mixing, diffusion due to concentration gradients, and adsorption-desorption due to the relative affinity of various solutes towards the fiber surface.

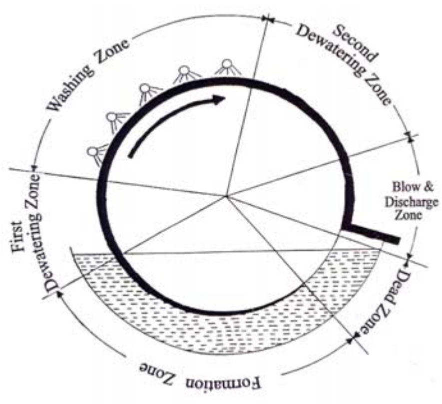

Figure 1: Schematic representation of a rotary vacuum washer

The mass transfer takes place from the fiber surface to the bulk fluid and *vice versa*, being further complicated by foaming and channeling.[1-5] The time allowed for diffusion is an important factor. Solute desorption also depends on the type of fibers, solute concentration, consistency and porosity.

Different investigators have proposed microscopic, macroscopic and semi-quantitative models for studying the washing behavior of the packed bed of porous particles. Numerous investigators[1,3,6-26] have contributed to the study of pulp washing. Brenner[7] and Sherman[8] have presented the general mathematical models for the displacement washing of both pulp fibers and spherical particles. Eriksson *et al.*,[1] Pellett,[9] Neretnieks[10-12] have provided the model incorporating the features of both axial dispersion and intrafiber diffusion. Grahs[13] has given a tri-zonal model to study the pulp washing phenomenon and Al-Jabari *et al.*[14] created the model for the adsorption of fillers onto fibers, to strengthen the paper. Edward and Rydin,[15] Wong and Reeve,[16] Kukreja *et al.*,[17] Potucek[5,18-21] have followed a dimensional axial dispersion model to study the washing of pulp fibers and of granular particles. Only a few investigators[17,22-26] have presented mathematical models for all zones of the rotary washer.

Most models are available for solid particles (spherical geometry), comparatively with pulp fibers (cylindrical geometry). The physical properties of fibers, such as consistency, compressibility, porosity, geometry, etc. are different in granular and in cylindrical particles, therefore a due weighing of these parameters should be made prior to developing a model for pulp fibers.

In the present study, the different zones of a rotary vacuum washer are subjected to a rigorous mathematical treatment. The models are expressed in terms of fundamental parameters. The output parameters are related to the efficiency parameters employed in paper industry to predict the system performance. A case study on a lab scale washer is provided, and the effect of some parameters on efficiency is also presented.

Models for the cake formation zone

Cake formation is highly dependent upon inlet vat consistency, drum fractional submergence, drum speed, pressure drop across the cake, pulp type and specific cake resistance.

Porosity (ε) of the packed bed is the ratio of the volume available for flow to the total volume. The hydrodynamics of filtration is highly influenced by the porous path through which the fluid moves. Porosity of the pulp suspension varies from position to position as the drum rotates. If assuming a negligible air effect, mean porosity can be defined as

$$\varepsilon = \frac{\rho_F (100 - C_y)}{\rho C_y + \rho_F (100 - C_y)} \quad (1)$$

Permeability (K), known as influencing considerably the fluid flow through porous media, is an important factor in cake filtration. It is a function depending essentially on the porosity and specific surface area of the fibers,[36] as follows:

$$K = \left[3.5\, a^2 (1-\varepsilon)^{3/2} \{1 + 57\, (1-\varepsilon)^3\} \right]^{-1} \quad (2)$$

The *angle of submergence* (θ), the angle subtended by the slurry level in the vat at the central axis, can be expressed as a function of the slurry level, drum radius and distance between the drum and the vat, as follows:

$$\theta = 2\cos^{-1}\left(\frac{R_0 + R - N_2}{R}\right) \quad (3)$$

Particularly, for pulp washing systems, θ varies from 120 to 150°, while R_0 varies from 9 to 10 cm. Also, the drum fractional submergence is given by $\psi = \theta / 2\pi$.

Cake thickness (L) represents the layer of fibers deposited on the outer surface of the drum, in transverse direction, due to vacuum. It is influenced by consistency, porosity, permeability, pressure drop across the cake and liquor viscosity. At constant porosity, permeability and negligible wire mesh resistance, the following static model[22] is used to calculate cake thickness:

$$\frac{dz}{dt} = \frac{K\Delta P \rho X_i}{z\eta\{(1-\varepsilon)\rho_F - \rho\varepsilon X_i\}} \quad (4)$$

where $X_i = C_{yi} / (1 - C_{yi})$.

By integrating for z varying from 0 to L and t from 0 to ψ / N, (ψ / N corresponds to cake formation time), one gets the following expression for L:

$$L^2 = \frac{2K\Delta P \rho \psi X_i}{N\eta\{(1-\varepsilon)\rho_F - \rho\varepsilon X_i\}} \quad (5)$$

The *filtrate flow rate* (V_f) is a complex function of the submergence angle, drum radius, pressure drop, frequency of drum rotation and properties of slurry and cake.

Modified Darcy's law for the volumetric flow of a liquid through a capillary of the circular cross-section is given by:

$$\frac{dV}{dt} = \frac{K\Delta P}{\eta z} \quad (6)$$

After some mathematical simplification, this equation gives the filtrate flow rate as:

$$V_f = NAL\{(1-\varepsilon)\rho_F - \varepsilon\rho X_i\} / \rho X_i \quad (7)$$

By considering filter medium resistance, V_f can be calculated[24] as:

$$V_f = \frac{R^2}{\psi}\left[\sqrt{\frac{NC'}{2+4Nt'}}\left\{\pi(1+4Nt')\sin^2\left(\frac{\theta}{2}\right) + \theta - \sin\theta\right\} - 2\pi N\sqrt{C't'}\sin^2\left(\frac{\theta}{2}\right)\right] \quad (8)$$

where $C' = 2K\Delta P\{(1-\varepsilon)\rho_F - \varepsilon\rho X_i\} / \eta\rho X_i$. When filter medium resistance is not considered, t' is taken to be zero.

The *fiber production rate* (*FPR*), the mass of cake produced per unit of time, depends on the area of the drum and on cake thickness. Assuming a negligible amount of solute adsorbed, it is given by

$$FPR = (1 - \varepsilon)\, NAL\, \rho_F \quad (9)$$

Models for the cake washing zone

The washing zone phenomenon is the sum of liquor displacement by the movement of the water plug controlled by fluid mechanics, dispersion due to back mixing, diffusion due to concentration gradient and adsorption-desorption due to the relative affinity of various solutes towards the fiber surface. Porosity, cake thickness, time of washing, liquor speed inside the cake pores and the amount of wash water applied are key variables for the washing zone. The non-ideal flow pattern is described by the dimensional axial dispersion model as:

$$D_L \frac{\partial^2 c}{\partial z^2} = u\frac{\partial c}{\partial z} + \frac{\partial c}{\partial t} + \frac{1-\varepsilon}{\varepsilon}\frac{\partial n}{\partial t} \quad (10)$$

The model gives importance to the following physical rate phenomena: axial dispersion of the bulk fluid, intrafiber Fick's law diffusion of the solute and liquid phase mass transfer, which accounts for the solute transport from the fiber surface to the flowing liquid.

Extensive research has been devoted to the linear adsorption isotherm,[1,7-10,16-18,22,37-39] which is widely followed, because it linearizes the differential equation describing the fluid flow behavior and reduces the mathematical complexities. In the linear adsorption isotherm, the deposition rate constant is in a forward direction, while the detachment rate constant is in a backward direction, both being of the first order, as follows:

$$\frac{\partial n}{\partial t} = k_1 c - k_2 n \quad (11)$$

Boundary and initial conditions

At the inlet of the bed, Danckwert's boundary condition has been applied:

$$uc - D_L \frac{\partial c}{\partial z} = uC_s \tag{12}$$

at $z = 0$ and $t > 0$, while at the outlet, to avoid the unacceptable conclusions that the fluid will pass through the maximum or minimum inside the bed, the concentration gradient is taken to be zero, *i.e.*:

$$\frac{\partial c}{\partial z} = 0, \tag{13}$$

at $z = L$ and $t > 0$.

Initially, it is assumed that the bulk fluid concentration is equal to the inlet solute concentration, *i.e.*:

$$c(z,t) = n(z,t) = C_i \tag{14}$$

for $0 < t < L/u$, where L/u corresponds to the displacement time, if assuming that pore tortuosity is equal to unity.

The model is converted into a dimensionless form with the dimensionless parameters presented in the nomenclature list. The resulting set of equations is solved by the Laplace transform. The detailed solution for the linear isotherm ($n = kc$) is available elsewhere.[25,26,39] The concentration of the solute c for any location and time is given by equation (15), where $Pe = \frac{uL}{D_L}$, $Z = \frac{z}{L}$, $T = \frac{ut}{L}$, $K' = \frac{k_1}{k_2}$, $G = \frac{k_2 L}{u}$, $H = \frac{(K'-1)C_s}{C_i - C_s}$ and β_n are the roots of $\beta \cot \beta + \frac{Pe^2 - 4\beta^2}{4Pe} = 0$ and p_n are the roots of $\beta = \frac{1}{2i}\sqrt{\frac{Pe^2(p+G) + 4Pep(p+G+\mu K'G)}{p+G}}$.

The *exit concentration* of the solute (C_e) leaving the bed at any time is obtained by setting Z = 1 in equation (15), represented by equation (16).

The *average concentration* of the solute (C_d) in the discharged pulp is obtained by equation (17).

The *mean concentration* of the filtrate (C_m) collected through the washing zone is found by equation (18).

The *filtrate flow rate through the washing zone* (V_w) is normally described by Darcy's law. In the present study, expressed in terms of mean concentration and average solute concentration, it evidences the relation between the concentration of black liquor solids in the discharge pulp and the filtrate flow rate as represented by equation (19).

$$\frac{c - C_s}{C_i - C_s} = \sum_{n=1}^{\infty} \frac{16Pe\beta_n^2 e^{p_n T} e^{PeZ/2}(p_n + G + \mu G - \mu GH)[Pe\sin\{(Z-1)\beta_n\} - 2\beta_n \cos\{(Z-1)\beta_n\}]}{p_n \sin\beta_n (p_n + G + \mu K'G)(Pe^2 + 4\beta_n^2)(Pe^2 + 4\beta_n^2 + 4Pe)\left[1 + \frac{\mu K'G^2}{(p_n + G)^2}\right]} \tag{15}$$

$$\frac{C_e - C_s}{C_i - C_s} = \sum_{n=1}^{\infty} \frac{-32Pe\beta_n^3 e^{p_n \tau} e^{Pe/2}(p_n + G + \mu G - \mu GH)}{p_n \sin\beta_n (p_n + G + \mu K'G)(Pe^2 + 4\beta_n^2)(Pe^2 + 4\beta_n^2 + 4Pe)\left[1 + \frac{\mu K'G^2}{(p_n + G)^2}\right]} \tag{16}$$

$$\frac{C_d - C_s}{C_i - C_s} = \int_0^1 \frac{c - C_s}{C_i - C_s} dZ = \sum_{n=1}^{\infty} \frac{-128Pe^2\beta_n^3 e^{p_n \tau} e^{Pe/2}(p_n + G + \mu G - \mu GH)}{p_n \sin\beta_n (p_n + G + \mu K'G)(Pe^2 + 4\beta_n^2)^2(Pe^2 + 4\beta_n^2 + 4Pe)\left[1 + \frac{\mu K'G^2}{(p_n + G)^2}\right]} \tag{17}$$

$$\frac{C_m - C_s}{C_i - C_s} = \frac{1}{T_W}\int_0^{T_W} \frac{c - C_s}{C_i - C_s} dt = \frac{L}{uT_W} + \sum_{n=1}^{\infty} \frac{32Pe\beta_n^3 e^{Pe/2}(p_n + G + \mu G - \mu GH)(e^{p_n} - e^{p_n \tau})}{\tau p_n^2 \sin\beta_n (p_n + G + \mu K'G)(Pe^2 + 4\beta_n^2)(Pe^2 + 4\beta_n^2 + 4Pe)\left[1 + \frac{\mu K'G^2}{(p_n + G)^2}\right]} \tag{18}$$

$$V_w = NAL\varepsilon[(C_i - C_d)/(C_m - C_s)] \tag{19}$$

Models for the cake dewatering zone

The major amount of liquor in the cake pores is removed in the washing zone. However, the small amount of liquor remaining in the cake pores is removed in the first and second dewatering zones. The degree of cake saturation is a key variable because of the simultaneous flow of liquor and air. For a two-phase flow through porous media, the following equation[23] is used:

$$\varepsilon L \frac{dS_s}{dt} = q - \frac{K\Delta P(S_e)^y}{\eta L} \quad (20)$$

However, in practice, real saturation S_s is used instead of effective saturation S_e. Equation (20) can be simplified with the relation $S_s = (S_e - 2S_eS_r + S_r)/(1 - S_eS_r)$, as follows:

$$S_e = [1 + 2K\Delta Pt_d / \eta\varepsilon L^2\{1 + (1 - S_r)^2\}]^{-1} \quad (21)$$

where $S_r = 0.025[K\Delta P / \sigma L]^{-0.26}$ (S_r is constant at constant ΔP).

The *filtrate flow rate through the dewatering zone* (V_d) can be calculated with the following expression:

$$V_d = NAL\varepsilon(1 - S_s) \quad (22)$$

Parameters for filter performance

The performance of an industrial pulp washing system, indicated by the quantity of black liquor solids removed with the amount of wash water added, can be classified into three categories *viz.*: wash liquor usage, solute removal and efficiency parameters. These parameters are linked to the fundamental ones, evaluated from the model, as shown below.

Wash liquor parameters

These parameters describe the amount of water used in pulp washing and measure the evaporator load. The amount of washing water should be kept as low as possible, otherwise the amount of liquor is to be treated in the evaporator, known as consuming excessive energy and thus increasing the costs and contributing to environmental decay.

The *dilution factor* (*DF*) of any stage is the difference between the wash liquor entering and the one leaving the washed pulp:

$$DF = L_s - L_d = 1 + (V_w\rho_s / FPR) - (100 / C_{yd}) \quad (23)$$

$$= \frac{(C_i - C_d)}{(C_m - C_s)} \frac{\varepsilon}{(1-\varepsilon)} \frac{\rho_s}{\rho_f} - \frac{(100 - C_{yd})}{C_{yd}} \quad (24)$$

The *wash liquor ratio* (*WR*) of any stage is the ratio of the wash liquor entering the liquor and the one leaving with the washed pulp:

$$WR = L_s/L_d = V_w\rho_sC_{yd} / [FPR(100 - C_{yd})] \quad (25)$$

$$= \frac{(C_i - C_d)}{(C_m - C_s)} \frac{\varepsilon}{(1-\varepsilon)} \frac{\rho_s}{\rho_f} \frac{C_{yd}}{(100 - C_{yd})} \quad (26)$$

The dilution factor and the washing ratio are related by the following relation:

$$DF = [(100 - C_{yd}) / C_{yd}](WR - 1) \quad (27)$$

When the volume of the wash liquor is equal to the volume of the liquor leaving with the washed pulp, then there is no excess of water nor of wash liquor, *i.e.* when $WR = 1$, $DF = 0$.

Solute removal parameters

These parameters, describing the amount of dissolved solids removed during a washing stage, can be used to predict the amount of bleach chemical consumption. Their value increases when the wash liquor usage parameters increase.

The *displacement ratio* (*DR*) of any stage is the ratio of the actual reduction of dissolved solids to their maximum possible reduction, as represented in equation (28):

$$DR = \frac{x_i - x_d}{x_i - x_s} = \frac{(C_i / \rho_i) - (C_d / \rho_d)}{(C_i / \rho_i) - (C_s / \rho_s)} \quad (28)$$

Obviously, the effect of sorption is not included in *DR*. It depends on the nature of the substance to be removed and hence it is a function of the solute present in the black liquor, being also influenced by pulping processes. Josephson *et al.*[27] have established the *DR* values for inorganic (Na salts) solutes using concentration measurements and organic (lignin) solutes, by UV analysis.

Efficiency parameters

Efficiency is directly proportional to the amount of solids removed during washing operations, depending mainly on four variables, namely, inlet vat consistency, discharge consistency, dilution factor and displacement ratio.

The *soda loss* (*SL*) during washing is expressed in terms of salt cake loss. Philips and Nelson[40] measured it in terms of Na_2SO_4, by the following relation:

$$SL = [(100 - \%WE)/100]\ \textit{salt cake charge} \quad (29)$$

where

$$\%WE = \left[1 - \frac{DF \cdot C_{yd} / (100 - C_{yd})}{[\{DF \cdot C_{yd} / (100 - C_{yd})\} + 1]^{MNEF+1} - 1}\right]100$$

The *washing efficiency* (*WE*) of any stage during washing operations represents the fraction of black liquor solids removed, being given by:

$$WE = \left[1 - \frac{(C_d - C_s)}{(C_i - C_s)} \frac{\rho_i}{\rho_s} \frac{(100 - C_{yd})}{(100 - C_{yi})} \right] \quad (30)$$

If assuming that the densities of all liquor streams entering and leaving the washer are the same, *DR* and *WE* can be correlated as follows:

$$WE = [1 - (1 - DR)(100 - C_{yd}) / (100 - C_{yi})] \quad (31)$$

Theoretically, when *DR* is 1, a 100% washing efficiency can be achieved, which is, nevertheless, an ideal stipulation which can not be realized in industry. *WE* and *WR* can be related as follows:

$$WE = \left[1 - \left\{ 1 - \frac{(C_m - C_s)}{(C_i - C_s)} \frac{(1-\varepsilon)}{\varepsilon} \frac{\rho_f}{\rho} \frac{(100 - C_{yd})}{C_{yd}} WR \right\} \frac{\rho_i}{\rho_s} \frac{(100 - C_{yd})}{(100 - C_{yi})} \right] \quad (32)$$

The washing efficiency of the entire washing system[41] can be calculated as:

$$WE_o = (L_b x_b - L_c x_c) / L_b x_b \quad (33)$$

In the case of a series of *n* washers, the overall *WE* and *DR* of each stage can be related as:

$$WE_o = [1 - (1 - DR_1)(1 - DR_2)\ldots(1 - DR_n)] \quad (34)$$

Norden's efficiency factor (*NEF*) is the number of mixing stages in series with complete mixing of underflow and overflow, required to achieve the same departing underflow and overflow values as those of the washing system without side streams, when the entering flows of the mixing stage system are the same as those of the washing system.[43] For a single stage, NEF can be written as:

$$NEF = \frac{\log\left(\dfrac{L_i (x_i - x_f)}{L_d (x_d - x_s)} \right)}{\log(L_s / L_d)} \quad (35)$$

When $NEF = 1$, the ideal mixing occurs while, when $NEF = \infty$, the plug flow results for solid particles liquor without leaching. In the case of a rotary drum, when the washing time is relatively short, it may be assumed that leaching contributes, even if to a low extent, to solute removal inside the fiber walls. When the amount of added washing water is equal to the amount of liquor present in the discharged pulp, *i.e.*, $WR = 1$, *NEF* is given by:

$$NEF = L_d (x_f - x_s) / L_i (x_i - x_d) \quad (36)$$

For the entire washing system, *NEF* can be found as:

$$NEF_o = \frac{\log\left(\dfrac{L_b (x_b - x_f)}{L_c (x_c - x_s)} \right)}{\log(L_s / L_d)} \quad (37)$$

The *modified Norden efficiency factor* (*MNEF*) is the number of ideal counter-current mixing stages equivalent to a washing system, operating at a standard discharge consistency (C_{yst}) of 10 or 12%, and the same dilution factor:[40]

$$MNEF = \frac{\log\left(\dfrac{L_i (x_i - x_f)}{L_d (x_d - x_s)} \right)}{\log(1 + DF / L_{st})} \quad (38)$$

where $L_{st} = (100\text{-}C_{yst})/C_{yst}$. Different types of washers or washers operating at the same *DF*, but over different consistency ranges, can be compared on the basis of their respective *MNEF* values. The *MNEF* of the entire system can be found by adding the value of each individual stage.

The *equivalent displacement ratio* (*EDR*) is used to compare the actual washer with a hypothetical one, operating at a standard inlet consistency of 1% and outlet of 12%. For the hypothetical washer,[41] *EDR* is calculated with the following formula:

$$(1 - EDR) = (1 - DR)(DCF)(ICF) \quad (39)$$

DCF = discharge correction factor = L_d /7.333

ICF = inlet correction factor = $99.0(L_i+DF)/[L_i(99.0+DF) - L_d(99.0-L_i)(1-DR)]$

The term equivalent means that the hypothetical washer has the same dilution factor as the actual one, the solid loss being the same in both cases. *EDR* is a useful mathematical tool for comparing washers of different designs. The washer with the highest *EDR* value for the same dilution factor will be the most effective one.

EXPERIMENTAL

The experiments were carried out on a single stage lab washer (radius – 23.46 and length – 30.48 cm). The combined length of the formation and first dewatering zones was of 60 cm, the lengths of the washing, second dewatering and discharge zones, plus the dead zone were of 30, 25 and 32.46 cm, respectively. The experiments were carried out on Indian hardwood pulp samples containing 81% eucalyptus, 17% bamboo and 2% pine. The pulp Kappa number was 20, the pH value of the vat liquor was 12 and the active alkali content in the vat liquor (as Na_2O) – 14.5%.

For the experiments, about 25-30 kg of unwashed pulp, blown at 348 K, were put in a feed tank (provided with a stirrer) and diluted with water at different consistencies. The cake was washed with fresh water. The frequency of drum rotation was adjusted with a knob. The fractional submergence of the drum was varied by placing plates in the pipe, for carrying the pulp overflow. The rotameters attached to the washing line were used to control and measure the wash water flow. Pressure drop varied independently, as due to the change produced in the degree of submergence, cake thickness, frequency of drum rotation and pulp consistency, being measured on the vacuum gauge mounted with the vacuum filter. At 20 ºC, the dynamic viscosity of the black liquor was determined in the laboratory, as 1.00×10^{-3} kg/ms. The surface tension of the black liquor and the density of the fibers were of 33 mN/m and 1560 kg/m^3, respectively. The black liquor density, known as depending on the content of solids, varied within 1011.8-1096.2 kg/m^3.

The input data are given in Table 1, while Table 2 lists all parameters measured during the experiments and Table 3 provides the performance parameters. The values of the mass transfer coefficients and of the longitudinal dispersion coefficient are taken from literature.[28] The ratio of interstitial velocity to longitudinal dispersion coefficient is taken[23,43] to be 1.30 cm^{-1}.

Table 1
Input data of different parameters in the experiments

Parameters	Range	Units
Inlet consistency of pulp (C_{yi})	0.53-1.99	%
Frequency of drum rotation	0.60-1.09	1/min
Wash water added (*WW*)	60-100	liters/h
Fractional submergence (ψ)	0.22-0.28	-
Concentration of solute (as Na_2O) inside the vat (C_i)	77.3-158.95	kg/m^3
Amount of liquor inside the vat (L_i)	49.14-187.18	kg/kg
Dissolved solids inside the vat ($x_i \times 10^2$)	7.64-14.50	kg/kg
Amount of wash water (L_S)	9.82-28.01	kg/kg

Table 2
Output data of some parameters in the experiments

Parameters	Range	Units
Outlet consistency of pulp (C_{yd})	3.85-11.85	%
Cake thickness (L)	0.0045-0.007	m
Pressure drop (ΔP)	3999-10664	Pa
Total porosity (ε)	0.954-0.986	-
Concentration of solids in discharge pulp (C_d)	18.36-38.97	kg/m^3

Dissolved solids in discharge pulp ($x_d \times 10^2$)	1.83-3.74	kg/kg
Amount of liquor in discharge pulp (L_d)	7.44-24.89	kg/kg
Filtrate flow rate ($V_f \times 10^5$)	5.59-13.89	m^3/s
Fiber production rate (*FPR*)	6.94-28.28	kg/s

Table 3
Some efficiency parameters

Parameters	Range	Units
Dilution factor (*DF*)	1.60-4.60	kg/kg
Wash ratio (*WR*)	1.14-1.57	-
Displacement ratio (*DR*)	0.74-0.80	-
Equivalent displacement ratio (*EDR*)	0.22-0.76	-
Washing efficiency (*WE*)	74.72-81.27	%
Norden efficiency factor (*NEF*)	2.5-3.8	-
Modified Norden efficiency factor (*MNEF*)	1.1-2.5	-

RESULTS AND DISCUSSION

Based on the above laboratory data, the effect of interstitial velocity, cake thickness, Peclet number, bed porosity and dilution factor on the performance evaluation parameters is discussed. In the graphs, linear trend lines are added for better understanding. The trend lines in the figures were derived by the least square method.

Effect of interstitial velocity: Interstitial velocity is inversely proportional to viscosity and porosity while, with the increase in interstitial velocity, the axial dispersion coefficient increases (Fig. 2). However, the rate of the increase depends on particle geometry and bed porosity. Potucek[21] also concluded that the dispersion coefficient is directly proportional to interstitial velocity.

Figure 3 illustrates that the increase in interstitial velocity has little effect on solute concentration in the discharged pulp. Previous investigations[35,44,45] have shown that, at low flow rates, washing efficiency is slightly dependent on interstitial velocity. Figure 4 shows that interstitial velocity has a meager effect on washing efficiency.

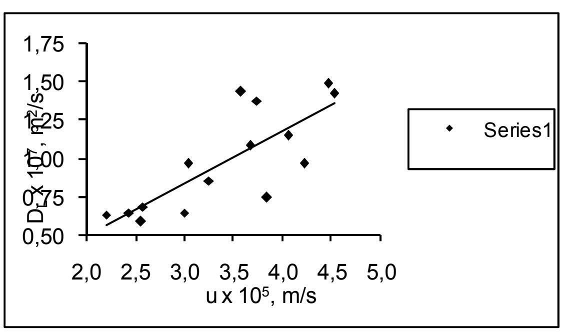

Figure 2: Effect of interstitial velocity on the axial dispersion coefficient

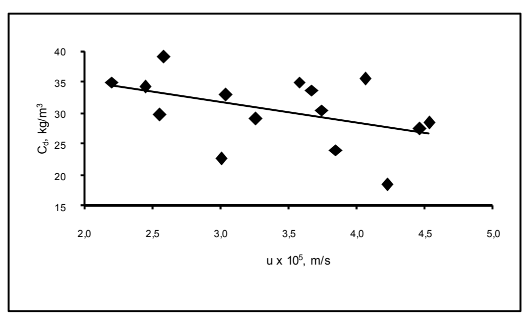

Figure 3: Effect of interstitial velocity on solute concentration in discharge pulp

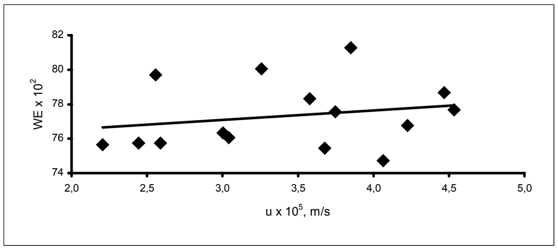

Figure 4: Effect of interstitial velocity on washing efficiency

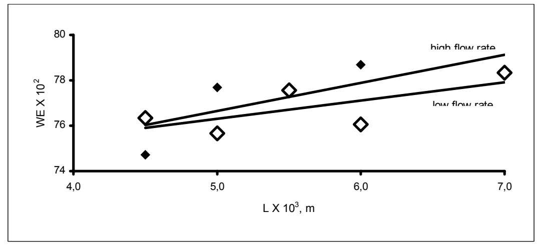

Figure 5: Effect of cake thickness on washing efficiency (high flow – upper line, low flow – lower line)

Effect of cake thickness: Figure 5 shows that the higher the cake thickness, the more solute will diffuse out of the cake pores and the better the washing operations will be. The increase in washing efficiency occurs for both low and high flow rates. Also, Lee[35] and Trinh[44] have demonstrated experimentally that an increase in cake thickness results in better washing.

Effect of the Peclet number: Figure 6 illustrates that the washing efficiency increases with the increase of the Peclet number, while Figure 7 indicates that the Peclet number increases when Norden's efficiency factor increases. These changes may be attributed to the axial dispersion coefficient, which decreases with the increase in the Peclet number; consequently, less back mixing occurs, thus leading to a better removal of the black liquor solids from the solute.

Effect of bed porosity: Porosity of the packed bed represents an important and sensitive physical factor, affecting brown stock washing. The higher particle porosity, the higher the permeability of the solute will be, the smaller the flow rates and the better the washing. Figure 8 shows that, as bed porosity increases, the displacement ratio also increases, which improves the washing operation. The results agree with the experimental ones.[35,44]

Effect of the dilution factor: The dilution factor and the displacement ratio are two typical parameters for determining the performance of washing systems. Figure 9 shows that any increase in the dilution factor causes an increase in the displacement ratio, a result similar to that reported by other investigators.[27,42,46] In actual practice, one has to maximize DR, while minimizing DF. Taking into account the evaporator steam costs, wash water cost, evaporator condensate savings, salt cake losses, organic losses and effluent treatment costs will optimize the dilution factor.[27]

Displacement ratio *vs* Norden efficiency factor: The displacement ratio involves no inlet and outlet pulp consistencies, therefore it does not really represent the effectiveness of a washer. This lacuna is solved by the Norden efficiency factor, which gives the ideal mixing stages necessary to achieve the same performance as in the washing stage.

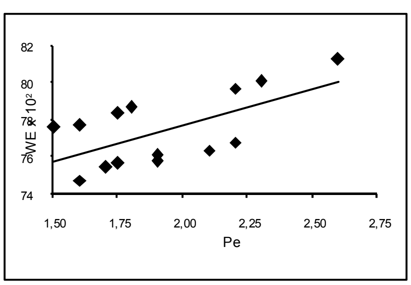

Figure 6: Effect of Peclet number on washing efficiency

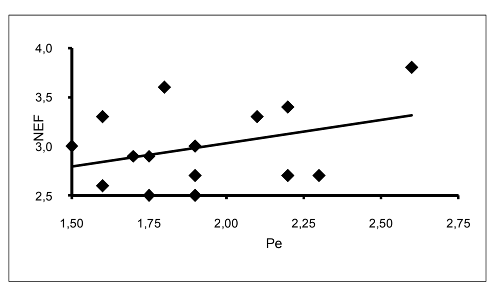

Figure 7: Effect of Peclet number on Norden efficiency factor

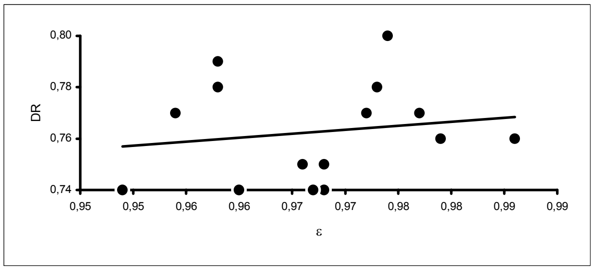

Figure 8: Effect of porosity on displacement ratio

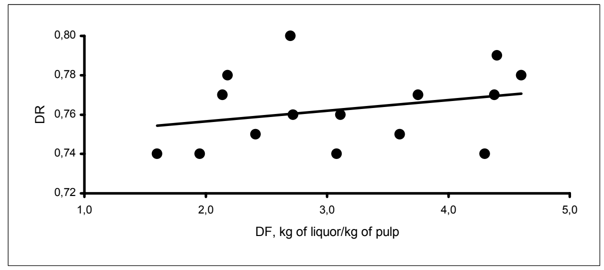

Figure 9: Effect of dilution factor on displacement ratio

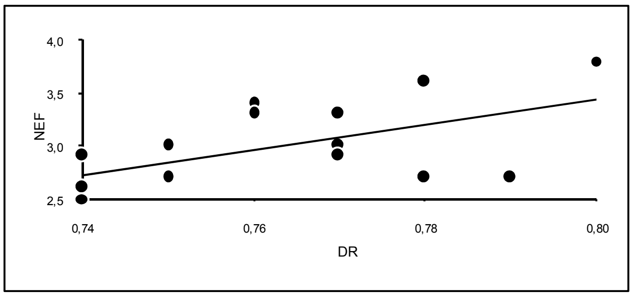

Figure 10: Displacement ratio *vs* Norden efficiency factor

It is influenced by pulp quality, shower distribution, specific loading, cake consistency, etc. Figure 10 illustrates the possible positive correlation between DR and NEF.

CONCLUSIONS

A mathematical model, microscopic in nature, for the packed bed of porous compressible particles (fibers) is provided. The cake formation zone is primarily dependent on vat consistency, liquor and pulp density, submergence of drum, speed of drum, pressure drop, pulp type and specific resistance of the cake. The washing zone is dominated by sorption effects, dispersion coefficient, mass transfer coefficients, porosity, permeability and cake thickness. These models can be extended to a system of washers working in a counter current manner, by means of a material balance. Laboratory data show that the dispersion coefficient increases by increasing interstitial velocity, whereas no effect was noticed on washing efficiency. Any increase in cake thickness and porosity was beneficial to washing. It may be observed that the real washing process is far from the ideal conditions of displacement washing, due to viscous fingering and axial diffusion or back mixing.

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NOMENCLATURE

a Specific surface of fibers, m^2/m^3
A Filtration area, m^2
c Solute concentration in the liquor, kg/m^3
C_e Exit concentration of solute, kg/m^3
C_i Concentration of solute inside the vat, kg/m^3
C_m Mean concentration of solute, kg/m^3
C_s Concentration of solute in the wash liquor, kg/m^3
C_y Pulp consistency, %
C_{yd} Discharged consistency of pulp, %
C_{yi} Inlet vat consistency of pulp, %
DF Dilution factor, kg of liquor/kg of pulp
D_L Longitudinal dispersion coefficient, m^2/s
DR Displacement ratio, dimensionless
EDR Equivalent displacement ratio, dimensionless
FPR Fiber production rate, kg/s
k_1, k_2 Mass transfer coefficients, 1/s
K Permeability constant of the cake, m^2

L Cake thickness, m
L_b Amount of liquor in the pulp coming from the blow tank, kg of liquor/kg of pulp
L_c Amount of liquor in the pulp leaving for the bleaching section, kg of liquor/kg of pulp
L_d Amount of liquor in the discharged pulp, $(100 - C_{yd})/C_{yd}$ kg of liquor/kg of pulp
L_i Amount of liquor inside the vat, kg of liquor/kg of pulp
L_s Amount of wash water, $V_w\rho_s/FPR$, kg of liquor/kg of pulp
n Concentration of solute on the fibers, kg/m^3
N Speed of the drum, rpm/60, 1/s
N_2 Level of slurry in the drum, m
NEF Norden's efficiency factor, dimensionless
ΔP Pressure drop, Pa
q Local shower flow (zero for the drying zone), m/s
R Radius of the drum, m
R_o Vertical distance between drum and vat, m
S_e Effective saturation, dimensionless
S_r Residual saturation, dimensionless
S_s Real saturation, dimensionless
SL Soda loss, kg/T OD pulp
t Time, s
t_d Time of the dewatering zone, s
t' Time to deposit a cake layer with a resistance equal to that of the filter cloth, s
T_w Total time of washing, s
u Interstitial velocity, m/s
V_d Filtrate flow rate through the cake dewatering zone, m^3/s
V_f Filtrate flow rate through the cake formation zone, m^3/s
V_w Filtrate flow rate through the cake washing zone, m^3/s
WE Washing efficiency, dimensionless
WR Wash liquor ratio, dimensionless
x_b Dissolved solids in the pulp coming from the blow tank, kg of solute/kg of liquor
x_c Dissolved solids in the pulp going to the bleaching section, kg of solute/kg of liquor
x_d Dissolved solids in the discharged pulp, kg of solute/kg of liquor
x_f Dissolved solids in the filtrate, kg of solute/kg of liquor
x_i Dissolved solids inside the vat, kg of solute/kg of liquor
x_s Dissolved solids in the wash liquor, kg of solute/kg of liquor
y Constant depending on particle size, dimensionless
z Variable cake thickness, m
ε Porosity of cake, dimensionless
η Viscosity of the liquor, kg/ms
θ Angle of submergence, Radian
ρ Density of water, kg/m^3
ρ_d Density of liquor in the discharged pulp, kg/m^3
ρ_F Density of fibers, kg/m^3
ρ_i Density of liquor inside the vat, kg/m^3
ρ_s Density of the wash water, kg/m^3
σ Surface tension of the liquor, mN/m
τ Dimensionless time, uT_w/L, dimensionless
ψ Fractional submergence of the drum, dimensionless

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